AFRL-RX-WP-TP-2012-0232

QUANTITATIVE EVALUATION FOR TWO-DIMENSIONAL ELECTROMAGNETIC PROPAGATION IN A DIELECTRIC MEDIUM USING MARKOV CHAIN MONTE CARLO METHOD (Preprint)

Jeremy S Knopp

Nondestructive Evaluation Branch
Metals, Ceramics, and NDE Division

Fumio Kojima

Kobe University
Organization of Advanced Science and Technology

FEBRUARY 2012

Approved for public release; distribution unlimited.

See additional restrictions described on inside pages

STINFO COPY

AIR FORCE RESEARCH LABORATORY
MATERIALS AND MANUFACTURING DIRECTORATE
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7750
AIR FORCE MATERIEL COMMAND
UNITED STATES AIR FORCE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
February 2012	Journal Article Preprint	01 November 2011 – 01 November 2011

4. TITLE AND SUBTITLE

QUANTITATIVE EVALUATION FOR TWO-DIMENSIONAL ELECTROMAGNETIC PROPAGATION IN A DIELECTRIC MEDIUM USING MARKOV CHAIN MONTE CARLO METHOD (Preprint)

5a. CONTRACT NUMBER
IN-HOUSE

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
62102F

6. AUTHOR(S)

Jeremy S Knopp (Metals, Ceramics, and NDE Division, Nondestructive Evaluation Branch (AFRL/RXLP))
Fumio Kojima (Kobe University, Organization of Advanced Science and Technology)

5d. PROJECT NUMBER
4349

5e. TASK NUMBER
40

5f. WORK UNIT NUMBER
LP111200

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Metals, Ceramics, and NDE Division
Nondestructive Evaluation Branch (AFRL/RXLP)
Materials and Manufacturing Directorate, Air Force Research Laboratory
Wright-Patterson Air Force Base, OH 45433-7750
Air Force Materiel Command, United States Air Force

Kobe University
Organization of Advanced Science and Technology
1-1 Rokkodai-cho, Nada-ku, Kobe
Hyogo 657-8501, Japan

8. PERFORMING ORGANIZATION REPORT NUMBER
AFRL-RX-WP-TP-2012-0232

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory
Materials and Manufacturing Directorate
Wright-Patterson Air Force Base, OH 45433-7750
Air Force Materiel Command
United States Air Force

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL/RXLP

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-RX-WP-TP-2012-0232

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES
PAO case number 88ABW-2011-5887, cleared 04 November 2011. The U.S. Government is joint author of this work and has the right to use, modify, reproduce, release, perform, display, or disclose the work. Submitted to the International Journal of Innovative Computing, Inform. Document contains color.

14. ABSTRACT

The motivation of this work is to quantify the degradation of aging electrical cables. The dielectric material parameter of insulation can be correlated with degradation. In this paper, the forward problem is posed as a microwave nondestructive evaluation (NDE) problem. A 2D finite difference time domain (FDTD) was used to evaluate the forward problem. The inverse problem is solved via a Bayesian approach. The Bayesian formulation describes the solution as a posterior distribution over the model parameters such as complex permittivity. Since there is no analytical solution for the posterior distribution, the Markov Chain Monte Carlo (MCMC) method is employed to numerically solve for it. The Metropolis-Hasting algorithm is used in particular. Results for computational experiments are demonstrated to show feasibility of this approach.

15. SUBJECT TERMS
Bayesian inversion, Markov Chain Monte Carlo, dispersive dielectric Bayesian inversion, Markov Chain Monte Carlo, dispersive dielectric

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
a. REPORT	b. ABSTRACT	c. THIS PAGE			Mark Blodgett
Unclassified	Unclassified	Unclassified	SAR	12	19b. TELEPHONE NUMBER *(Include Area Code)* N/A

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18

Quantitative Evaluation for Two-dimensional Electromagnetic Propagation in a Dielectric Medium using Markov Chain Monte Carlo Method

Fumio Kojima[1] and Jeremy S Knopp[2]
[1]Organization of Advanced Science and Technology, Kobe University
1-1 Rokkodai-cho, Nada-ku, Kobe, Hyogo 657-8501, Japan
E-mail: kojima@koala.kobe-u.ac.jp
[2] AFRL/RXLP, Nondestructive Evaluation Branch
Air Force Research Laboratory
Wright-Patterson AFB, OH 45433 USA

Abstract

The motivation of this work is quantify the degradation of aging electrical cables. The dielectric material parameter of insulation can be correlated with degradation. In this paper, the forward problem is posed as a microwave nondestructive evaluation (NDE) problem. A 2D finite difference time domain (FDTD) was used to evaluate the forward problem. The inverse problem is solved via a Bayesian approach. The Bayesian formulation describes the solution as a posterior distribution over the model parameters such as complex permittivity. Since there is no analytical solution for the posterior distribution, the Markov Chain Monte Carlo (MCMC) method is employed to numerically solve for it. The Metropolis-Hasting algorithm is used in particular. Results for computational experiments are demonstrated to show feasibility of this approach.

1 Introduction

The degradation of insulation in electrical cables used with instruments and control facilities is of serious concern for management of aging large scale systems such as airplanes, power plants, etc [1]. The fact that dielectric properties of materials used in electrical cables change as they degrade over the course of their service life provides a means of quantifying the degradation if the dielectric property can be measured accurately. Microwave NDE can make these measurements and thus quantitatively assess the degradation of cable insulation. The idea has been already applied to the in-service inspection in a variety of industrial applications. Time-Resolved Microwave Dielectric Absorption (TRMDA) enables the measurement of the evanescent electrical field leaking from the probe into the cable insulation by using cavity resonator [2]. This electrical field changes as a function of the dielectric properties of material, which in turn changes the resonant frequency. Figure 1 depicts the overall configuration of the current nondestructive testing setup. The nondestructive test is performed by evaluating a bunch width of reflected microwave. This is because the increasing the whole of cavity in cable insulation can be detected as variation of the bunch width. Although the method has been a conventional method for characterizing aging properties, there are several practical disadvantages. One is that the detection and characterization of variation of bunch width are not feasible for on-site inspection. Also continuous surveillance of the variation is very crucial. Taking this into account, our concern in this paper is to develop a computational method for identifying dielectric properties of electrical cables based on a microwave measurement system.

The measurement of material aging parameters continues to be a very challenging problem. In our approach, we formulate a forward problem arising in micro wave NDE. More specifically, it represents a mathematical model of a specific measurement using the parameter-to-output mapping with the appropriate admissible class of material parameters, To this end, the mathematical model is described by Maxwell's equations in 2D. The numerical scheme for solving the forward problem is essentially the finite-difference time-domain method (FDTD) in two spatial dimensions. Thus, the inversion of material parameters is implemented with the aid of the forward problem. The nonlinear least square identification (OLSI) is a conventional inverse methodology and there have been many efforts in a variety of industrial applications. For OLSI related to electromagnetic inversion, we refer to [3][4][5][6][7]. However,it is well known that the problem mentioned above has many solutions due to the fact it is ill-posed. Recently, interest has grown in stochastic inversion using Markov Chain Monte Carlo (MCMC) methods [8][9]. Previous efforts based on a similar approach has been proposed [10][11]. The method has

great advantages for the more practical aspects of inversions such as setting initial guesses and overcoming local minimums of the inverse solution. In this paper, a stochastic inversion technique based on the Metropolis-Hasting algorithm is applied to the problem presented here.

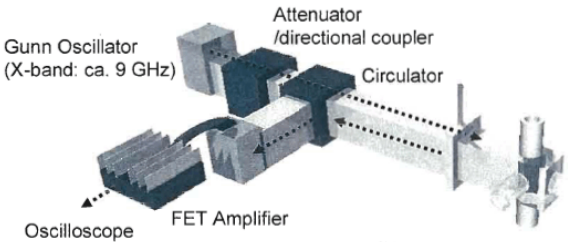


Fig. 1: Nondestructive test using TRMDA

2 Forward Analysis for NDE

2.1 Problem Domain

For the simplicity of discussions, we consider a two dimensional bounded domain $x = (x_1, x_2) \in \Omega \subset R^2$ as an inspection area. Let Ω_c be a cable domain defined on Ω. The cable domain consists of a material Ω_d and a metal core Ω_o. The material corresponds to a polymer of which the dielectric constant ϵ_r and conductivity σ_d often vary at different frequencies. Figure 2 depicts the two-dimensional problems space considered here. Our electromagnetic interrogation is then

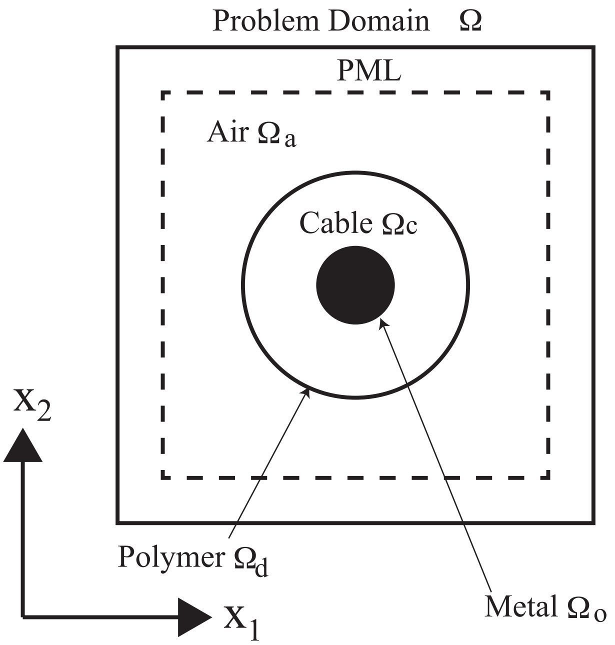


Fig. 2: Schematic diagram of geometry

represented by electric and magnetic fields in Ω. Let $D_3(t,x)$, $H_1(t,x)$, $H_2(t,x)$ and $E_3(t,x)$ be the component of electric flux density, magnetic and electric field at time $t \in [0,T]$ and at location $x \in \Omega$. In order to use Gaussian units, we also use the normalizing terms,

$$\tilde{D}_3 := \sqrt{\frac{1}{\epsilon_0 \mu_0}} D_3$$

$$\tilde{E}_3 := \sqrt{\frac{\epsilon_0}{\mu_0}} E_3$$

where ϵ_0 and μ_0 are the permittivity and magnetic permeability of air, respectively. From Maxwell's equations, the electromagnetic propagation in the transverse magnetic (TM) mode is governed by

$$\frac{\partial \tilde{D}_3(t,x)}{\partial t} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \left(\frac{\partial H_2(t,x)}{\partial x_1} - \frac{\partial H_1(t,x)}{\partial x_2} \right) + J_s(t,x) \quad \text{in } [0,T] \times \Omega \tag{1}$$

$$\tilde{D}_3(\omega,x) = \epsilon_r^*(\omega) \tilde{E}_3(\omega,x) \tag{2}$$

$$\frac{\partial H_1(t,x)}{\partial t} = -\frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{\partial \tilde{E}_3(t,x)}{\partial x_2} \quad \text{in } [0,T] \times \Omega \tag{3}$$

$$\frac{\partial H_2(t,x)}{\partial t} = -\frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{\partial \tilde{E}_3(t,x)}{\partial x_1} \quad \text{in } [0,T] \times \Omega \tag{4}$$

with zero initial states and with the absorption boundary conditions. The complex relative dielectric constant ϵ_r^* of the polymer Ω_d can be represented by a lossy dielectric medium of the form;

$$\epsilon_r^* = \epsilon_r + \frac{\sigma_d}{j\omega\epsilon_0}. \tag{5}$$

On the other hand, The value of this parameter in $\Omega_a = \Omega - \Omega_c$ becomes $\epsilon_r^* = 1$. The value of that in Ω_o is also given by

$$\epsilon_r^* = 1 + \frac{\sigma_o}{j\omega\epsilon_0} \tag{6}$$

where σ_o denotes the conductivity of the metal core. The source current J_s is the test signal in our inspection process and is given by

$$J_s(t,x) = \delta(\Omega_s) g_s(t) I_{(0,t_s)}(t) \tag{7}$$

where δ denote the array of actuator in Ω such as $\Omega_s \bigcap \Omega_c = \emptyset$. In Eq. (7), the test signal is truncated at a finite time t_s by the indicator function I. Detections can be made through the allocation of sensor arrays Ω_o in Ω by

$$Y(t,x;\mathbf{q}) = E_3(t,x;\mathbf{q}) \quad \text{in } \Omega_o \tag{8}$$

where the parameter vector $\mathbf{q}$ to be identified is denoted by

$$\mathbf{q} = \{\epsilon_r, \sigma_d\}. \tag{9}$$

Thus the forward problem for our NDE is formulated as

$$\mathbf{q} \Longrightarrow Y(t,x;\mathbf{q}). \tag{10}$$

2.2 Numerical Scheme

The finite-difference time-domain method (FDTD) in two spatial dimensions can be implemented to solve the forward model [12]. Putting Eqs.(1), (3), and (4) into the finite difference scheme results in the following difference equations:

$$
\begin{aligned}
&\frac{D_3^{n+1/2}[i,j] - D_3^{n-1/2}[i,j]}{\Delta t} \\
&= \frac{1}{\sqrt{\epsilon_0\mu_0}} \left\{ \frac{H_2^n[i+1/2,j] - H_2^n[i-1/2,j]}{\Delta x_1} \right. \\
&\quad \left. \frac{H_1^n[i,j+1/2] - H_1^n[i,j-1/2]}{\Delta x_2} \right\} \\
&\frac{H_1^{n+1}[i,j+1/2] - H_1^n[i,j+1/2]}{\Delta t} \\
&= -\frac{1}{\sqrt{\epsilon_0\mu_0}} \left\{ \frac{E_3^{n+1/2}[i,j+1] - E_3^{n+1/2}[i,j]}{\Delta x_2} \right\} \\
&\frac{H_2^{n+1}[i,j+1/2] - H_2^n[i,j+1/2]}{\Delta t} \\
&= -\frac{1}{\sqrt{\epsilon_0\mu_0}} \left\{ \frac{E_3^{n+1/2}[i+1,j] - E_3^{n+1/2}[i,j]}{\Delta x_1} \right\}
\end{aligned}
$$

In order to prevent spurious reflections from the edge of the problem space, the perfectly matched layer (PML) is adopted to two dimensional domain (See [13] for more details). The principle of micro wave testing can be represented by the simulation of plane waves. Since the propagating plane wave must not interact with the absorbing boundary conditions like PML, the problem space can be divided up into two regions such as total fields and scattered field. Figure 3 illustrates how the incident plane wave behaves. To simulate a plane wave interacting with a cable object, the cable object according to its electromagnetic properties must be specified. The simulation of a plane wave pulse hitting a dielectric cylinder with $\epsilon_r = 30$ and $\sigma = 0.3[m/s]$ is demonstrated in Fig. 4. After the time step $T = 50$, the pulse is interacting with the cylinder. At time step $T = 75$ some of it passes through the cylinder while the other part of it goes around that. It could be also recognized that, at $T = 100$, the main part of the propagation is being subtracted out the end of the total field.

3 Stochastic Inverse Analysis

Our problem is to identify dielectric parameters related to cable insulation

$$\mathbf{q} = \{\epsilon_r, \sigma_d\}$$

using the information obtained through a scattering experiment for the forward problem stated in the previous section. The observations consist of sampling data associated with the electrical field. We suppose that the experimental observations consist of the values of electrical field at the measurement point $x_p = \{x_p^1, x_p^2\}$ at

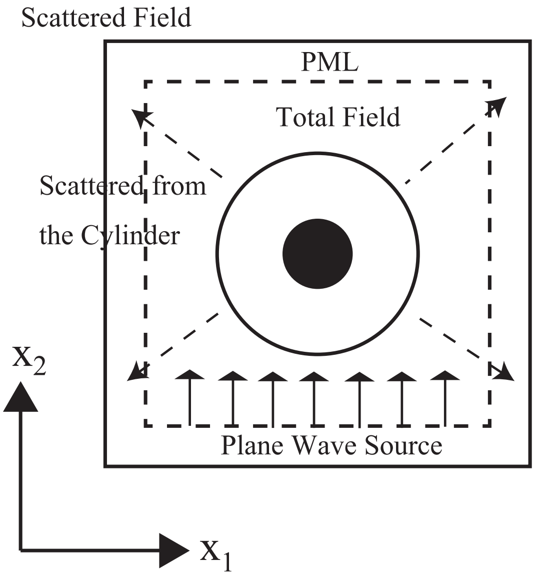


Fig. 3: Diagram of a plane wave hitting a cable object

the sampling time interval Δt. Let $\mathbf{Y}_d^N = \{\hat{Y}_d[n]\}_{n=1}^N$ denote the collection of the data we seek to reconstruct and $\mathbf{Y}^N(\mathbf{q}) = \{Y(n\Delta t, x_p; \mathbf{q})\}_{n=1}^N$ be the electrical field arising from the forward problem with dielectric parameter vector $\mathbf{q}$. Then the structural equation model is formulated as

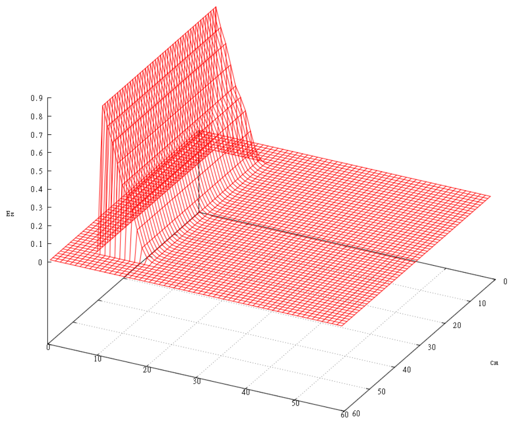
$$
\begin{aligned}
\hat{Y}_d[n] &= Y(n\Delta t, x_p; \mathbf{q}) + \eta_n, \quad \eta_n \sim \mathcal{N}(0, \tau^2), \\
&\qquad (n = 1, 2, \cdots N).
\end{aligned} \tag{11}
$$

where η_n denotes the mutually independent sequence of additive measurement noise. With the background knowledge of Bayes formula, the posteriori density function with respect to the set of unknown parameter vector $\mathbf{q}$ can be simply written by

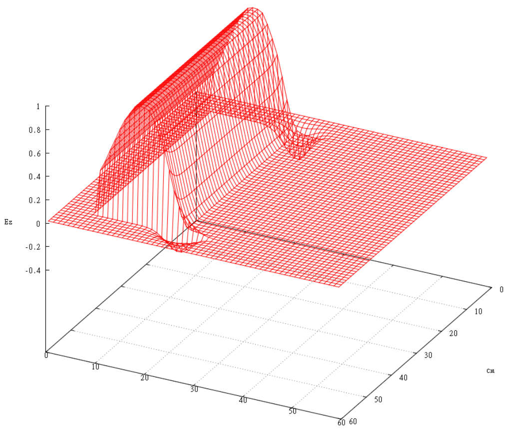
$$
\begin{aligned}
p(\epsilon_r|\sigma, \mathbf{Y}^N) &\propto p(\epsilon_r) \prod_{n=1}^N \frac{1}{\sqrt{2\pi}\tau} \\
&\quad \exp\left(-\frac{\left| Y(n\Delta t, x_p; \mathbf{q}) - \hat{Y}_d[n] \right|^2}{2\tau^2} \right)
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
p(\sigma_d|\epsilon_r, \mathbf{Y}^N) &\propto p(\sigma) \prod_{n=1}^N \frac{1}{\sqrt{2\pi}\tau} \\
&\quad \exp\left(-\frac{\left| Y(n\Delta t, x_p; \mathbf{q}) - \hat{Y}_d[n] \right|^2}{2\tau^2} \right)
\end{aligned} \tag{13}
$$

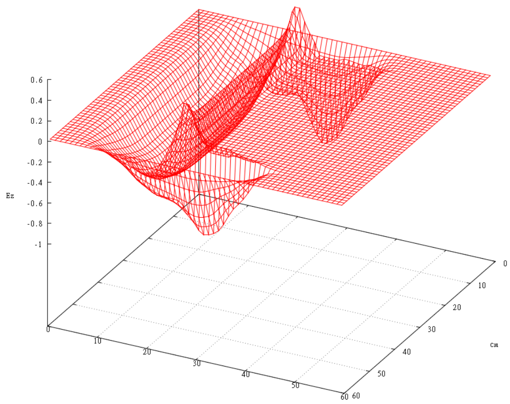
where $p(\epsilon_r)$ and $p(\sigma_d)$ denote a priori density functions of ϵ_r and σ_d. An estimation algorithm can be performed by sampling procedures for the posteriori distribution given by Eqs. (12) and (13) from which sample paths



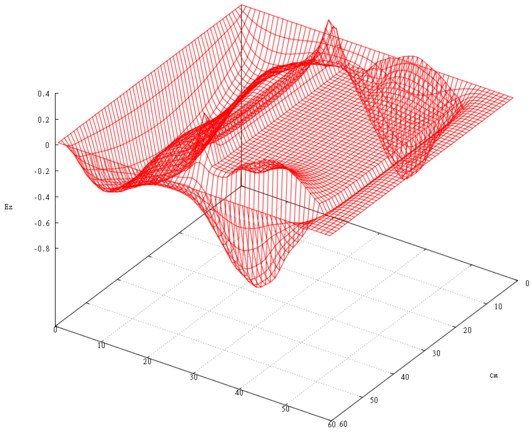
(a) $T = 25$



(b) $T = 50$



(c) $T = 75$



(d) $T = 100$

Fig. 4: Simulation of a plane wave pulse

can be drawn using Markov chains. For the practical implementation of MCMC, Metropolis-Hasting algorithm is applied to the problem considered here.

4 Experimental Results

To test the feasibility of the estimation approach, we produce synthetic data for the observations $\mathbf{Y}^N = \{Y[n]\}_{n=1}^N$ by adding random noise to the results of the simulation with a known set of parameters $\mathbf{q}^{true} = \{\epsilon_r^{true}, \sigma_d^{true}\}$. The data to be identified are generated with the forward problem in the previous section using the following parameter values:

$$\begin{aligned} \epsilon_r^{true} &= 30.0 \\ \sigma_d^{true} &= 0.3 \quad [S/m] \end{aligned}$$

The carrier frequency of the plane wave is $90GHz$ and the duration of sampling time is taken as $\Delta t = [s]$, respectively. The observation data consist of 512 measurements of the electric field taken at $x_p = (0.5, 0.5)$. Figure 5 depicts the model data with true parameters. A *priori* densities of ϵ_r and σ are preassigned as normal distributions;

$$\begin{aligned} p(\epsilon_r) &= \frac{1}{\sqrt{2\pi}\tau_r} \exp\left(-\frac{\left|\epsilon_r - \epsilon_r^0\right|^2}{2\tau_r^2}\right) \\ p(\sigma_d) &= \frac{1}{\sqrt{2\pi}\tau_\sigma} \exp\left(-\frac{\left|\sigma - \sigma^0\right|^2}{2\tau_\sigma^2}\right) \end{aligned}$$

where ϵ_r^0 and σ^0 denote the nominal values of dielectric parameters. The artificial noise term was provided

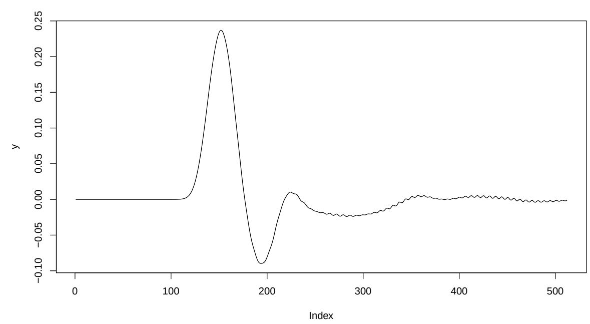
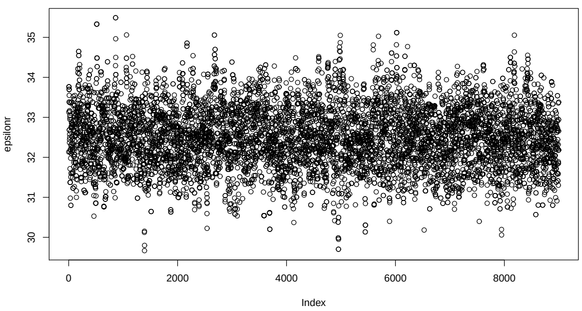
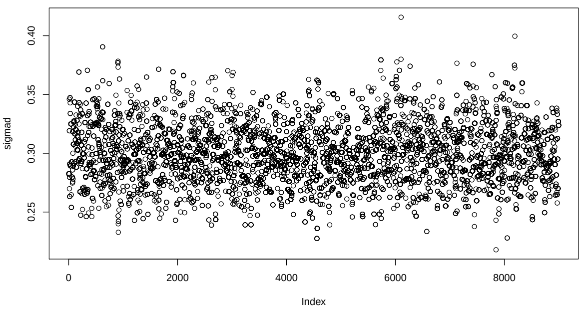


Fig. 5: Synthetic data with true parameter values (noise free)

using a standard Gaussian random generator $N(0, 0.1)$. After 10000 iterations and discarding the first 1000 iterations, the posterior means of $\mathbf{q}$ were evaluated. Typical paths based on 9000 draws are presented in Fig. 6. The resulting marginal posteriors appear in Fig. 7. Estimation results are summarized in Table 1.

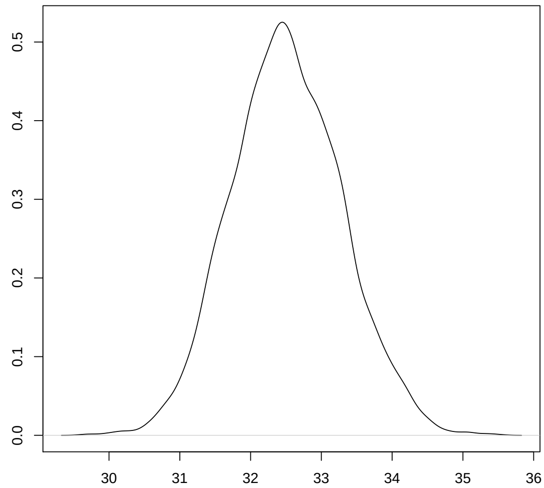


(a) ϵ_r

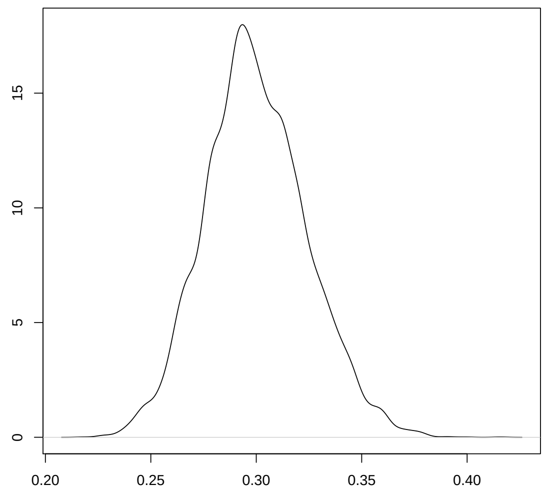


(b) σ_d

Fig. 6: Trajectory of the chains



(a) ϵ_r



(b) σ_d

Fig. 7: Marginal posteriori density estimates

Table 1: Estimation summary

Quantities	ϵ_r	$\sigma_d[S/m]$
True	30.00	0.3
Min	29.67	0.2179
1st Quater	31.98	0.2838
Median	32.49	0.2986
Mean	32.51	0.3000
3rd Quarter	33.04	0.3154
Max	35.49	0.4157

5 Conclusions

Microwave analysis was considered for the detection and the characterization of cable insulation. The forward problem in 2D was considered by constructing deterministic parameter-to-output mappings based on the frequency dependent media with material quantities of cable insulation. A numerical scheme was developed using the FDTD method in order to solve the forward problem. Secondly, stochastic inverse methodology was considered with the background knowledge of Bayes formula. The full probability model was specified by the deterministic formula treated in the forward problem and by the stochastic relationship for their measurement strategies. The posteriori density function was provided with likelihood functional update based on iterative procedures of measurements and with a priori probability densities with respect to the unknown quantities. The validity and feasibility of our proposed method were demonstrated through computational experiments for appropriate specific examples. The MCMC samplers using Metropolis-Hasting algorithm was tested for our inverse problem.

Acknowledgement

This material is based on research sponsored by AOARD, under agreement number No. FA2386-10-1-4145.

References

[1] Japan Nuclear Energy Safty Organization (JNES), *Technical evaluation review manuals for aging management*, 2005. (in Japanese)

[2] A.R. Santhamurthy, T.A.P. Rao, J. Sobhanadri, and V.R. Murthy, Measurement of dipole moments and lifetimes of triplet states of fluorenone and its derivatives by time resolved microwave dielectric absorption, *JSPJ*, **67** (1998), 1220-1225.

[3] H.T. Banks, F. Kojima, and W.P. Winfree, Boundary estimation problems arising in thermal tomography, *Inverse problems*, vol.6, no.4, pp.897-921, 1990.

[4] H.T. Banks, M.W. Buksas, and T. Lin, *Electromagnetic Material Interrogation using Conductive Interfaces and Acoustic Wavefronts*, Frontiers in Applied Mathematics, SIAM Publishers, Philadelphia, 2000.

[5] H.T. Banks and F. Kojima, Identification of material damage in two-dimensional domains using SQUID-based nondestructive evaluation system, *Inverse Problems*, vol.18, no.6, pp.1831-1856, 2002.

[6] F. Kojima, Inverse problems related to electromagnetic nondestructive evaluation, In *Research Directions in Distributed Parameter Systems*, Frontiers in Applied Mathematics, SIAM Publisher, Philadelphia, pp.219-236, 2003.

[7] F. Kojima and A. Ausri, Identification of stress corrosion cracking of SUS samples arising in electromagnetic nondestructive testing, *J. Inv. Ill-posed Problems*, vol.15, no.8, pp.799-812, 2007.

[8] W.R. Gilks. S. Richardson and D.J. Spiegelhaltger, *Markov Chain Monte Carlo in Practice*, Chapman & Hall/CRC, New York 1996.

[9] D. Gamerman, *Markov Chain Monte Carlo, Stochastic simulation for Bayesian inference*, Chapman & Hall, New York 1997.

[10] F. Kojima and S. Kamezaki, Identification of defect profiles using a inspection model and informative distributions, *Proc. 35th ISCIE International Symposium on Stochastic Systems Theory and Its Applications*, ISCIE (2004), 235-240.

[11] F. Kojima, Inverse problem for electromagnetic propagation in a dielectric medium using Markov Chain Monte Carlo Method, *Proc. 42th ISCIE International Symposium on Stochastic Systems Theory and Its Applications*, ISCIE (2011).

[12] D. M. Sullivan, Electromagnetic Simulation using the FDTD Method, IEEE Press, New York 2000.

[13] J.P. Berenger, A perfectly matched layer for the absorption of electromagnetic waves, Journal of Computational Physics, vol.114, pp. 185-200, 1994.